SF3961 Statistical Inference

2015/16

Homework 3

PROBLEM 1: Problem III.3, p. 208 in Schervish. (20%)

PROBLEM 2: Problem III.4, p. 208 in Schervish. (20%)

PROBLEM 3: Problem III.9, p. 209 in Schervish. (20%)

PROBLEM 4: Let $\Omega = \{0, 1\}$, $\aleph = \{1, 2, 3, 4, 5\}$, and the loss function

$$\begin{array}{lllll} L(0,1) = 0, & L(0,2) = 1, & L(0,3) = 0.8, & L(0,4) = 0.2, & L(0,5) = 1, \\ L(1,1) = 1, & L(1,2) = 0, & L(1,3) = 0.1, & L(1,4) = 0.6, & L(1,5) = 1. \end{array}$$

(a) Draw the risk set and display all admissible rules.
(b) Show that there is a minimax rule and find it and illustrate the corresponding risk function in your figure.
(c) Determine the least favorable prior and illustrate it in your figure.
(d) Find all Bayes rules with respect to the least favorable prior and illustrate the corresponding risk functions in your figure. (20%)

PROBLEM 5: Consider the following situation. You have an amount of m dollars to bet on the outcome of a Bernoulli random variable X_{n+1}. You observe $X = (X_1, \ldots, X_n)$. Suppose $X_1, \ldots, X_{n+1}$ are conditionally iid Ber(θ) random variables given $\Theta = \theta$. Based on the observations in X you have to make a decision whether to bet on $X_{n+1} = 0$ or $X_{n+1} = 1$. If you win, you gain the amount m and otherwise you loose m.

(a) Formulate this as a Bayesian decision problem. Write down the sample space $\mathcal{X}$, the parameter space Ω, and the action space $\aleph$. Choose an appropriate prior distribution and an appropriate loss function of your choice. Then find the best decision rule, i.e. the decision rule δ that minimizes the posterior risk simultaneously for all x.

(b) Formulate this as a classical decision problem. Can you characterize the admissible decision rules with the help of Neyman-Pearson? (20%)

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